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References

[1] H. A. Wheeler, "The transmission-line properties of a round wire, between parallel planes," *IRE Trans. Antennas Propagat.*, vol. AP-3, pp. 203–207, Oct. 1955.

[2] E. G. Cristal, "Coupled circular cylindrical rods between parallel ground planes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, July 1964.

[3] B. N. Das and K. V. Seshagiri Rao, "Analysis of an elliptical conducting rod between parallel ground planes by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1079–1085, July 1982.

[4] B. N. Das, K. V. Seshagiri Rao, and A. K. Mallick, "Analysis of an oval symmetrically located inside a rectangular boundary by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 403–406, May 1983.

[5] K. J. Binns and P. J. Lawrenson, *Analysis and Computation of Electric and Magnetic Field Problems*. New York: Pergamon Press, 1963.

[6] P. F. Byrd and M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Scientists*. New York: Springer, 1971.

[7] J. C. Tippet and D. C. Chang, "Radiation characteristics of electrically small devices in a TEM transmission cell," *IEEE Trans. Electromagn. Compat.*, vol. EMC-18, no. 4, pp. 134–140, Nov. 1976.

[8] J. S. Rao and B. N. Das, "Analysis of asymmetric stripline by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 299–303, Apr. 1979.

Spectral Domain Analysis of Interacting Microstrip Resonant Structures

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***Abstract*—An analysis of the interacting microstrip resonant structures, namely, the half-wave coupled and the quarter-wave coupled rectangular microstrip resonators is performed with the hybrid-mode formulation of the spectral domain technique. The resonant frequencies in the even and odd resonance modes are evaluated from the numerical solution of the characteristic equation. Results agree within ±1.5 percent of the experimental values.**

I. Introduction

Among various types of interacting resonant structures in microwave integrated circuit applications, the half-wave coupled and the quarter-wave coupled rectangular microstrip resonators are extensively used as network elements. In such structures, the propagation of waves is described in terms of the even and odd modes [1]. There is some difference between the even- and odd-mode phase velocities at lower microwave frequencies, but, as the frequency of operation increases, the divergence in the even- and odd-mode phase velocities becomes quite significant. In directional couplers, this causes degradation of match, directivity and isolation. It causes spurious response and reduces the operating bandwidth of filters. There have been many attempts, in the past, to investigate the effect of unequal phase velocities and to achieve their equalization [2]–[4].

In the above context, the study of interacting rectangular microstrip resonators assumes considerable importance. The early experimental work by Easter and Ritchings [5] provide some background information to assess the predictability of resonant frequencies in the even and odd resonance modes of a quarter-wave coupled rectangular microstrip resonator. An exact analysis of interacting resonant structures has been considered intrinsically difficult [6] and, therefore, there has not been any attempt to determine the resonant frequencies of the abovementioned structures. Consequently, various attempts to alleviate the effect of unequal phase velocities on the performance of the directional couplers and filters utilized the available information on the coupled microstrip lines.

In this paper, we have utilized the hybrid-mode formulation in the spectral domain to analyze these resonant structures. The divergence in the even- and odd-mode phase velocities of the abovementioned structures is thus determined in terms of their resonant frequencies.

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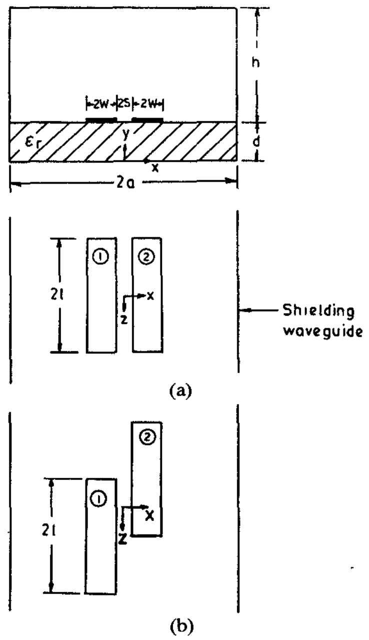

Fig. 1. Interacting rectangular microstrip resonators: (a) half-wave coupled parallel rectangular microstrip resonator; (b) quarter-wave coupled parallel rectangular microstrip resonator.

II. Analysis

The interacting rectangular microstrip resonant structures in a shielding waveguide configuration are shown in Fig. 1. The basic building block in each case is a rectangular microstrip resonator of length $2l$ and width $2w$. The shielding waveguide has dimensions $2a$ and $d+h$. The dielectric substrate of relative permittivity ϵ_r has thickness d above the ground plane.

In the spectral domain analysis of the structure [7]–[11], the Fourier transform of the dyadic Green's functions are related to the transforms of the current densities on the conductors and the electric fields in the region of the interface complementary to the conductors, via the equation

$$\begin{bmatrix} \tilde{G}_{11}(\hat{k}_n,\beta,k_0) & \tilde{G}_{12}(\hat{k}_n,\beta,k_0) \\ \tilde{G}_{21}(\hat{k}_n,\beta,k_0) & \tilde{G}_{22}(\hat{k}_n,\beta,k_0) \end{bmatrix} \begin{bmatrix} \tilde{J}_{xc}(\hat{k}_n,\beta) \\ \tilde{J}_{zc}(\hat{k}_n,\beta) \end{bmatrix} = \begin{bmatrix} \tilde{E}_{zc}(\hat{k}_n,\beta) \\ \tilde{E}_{xc}(\hat{k}_n,\beta) \end{bmatrix} \tag{1}$$

where $\hat{k}_n$ and β are the Fourier transform variables, and k_0 is the free-space wavenumber. $\tilde{J}_{xc}$, $\tilde{J}_{zc}$, $\tilde{E}_{xc}$, and $\tilde{E}_{zc}$ are the current densities on the interacting conductors and electric fields in the region of the interface complementary to the conductors, respectively. $\tilde{G}_{11}$, $\tilde{G}_{12}$, $\tilde{G}_{21}$, and $\tilde{G}_{22}$ are the transformed Green's functions. With the application of Galerkin's procedure and Parseval's theorem, we obtain a set of algebraic equations in terms of the unknown constants of the basis functions. For a given structure, a nontrivial solution of the wavenumber k_0 is obtained by setting the determinant of the coefficient matrix equal to zero and finding the root of the equation.

III. Current Density Functions

The current density distribution functions on an interacting rectangular microstrip resonator can be easily obtained from that of its building block, the rectangular microstrip resonator. If $\tilde{J}_z$ and $\tilde{J}_x$ are the longitudinal and transverse current densities, respectively, on a rectangular microstrip resonator, then the corresponding current densities $\tilde{J}_{zc}$ and $\tilde{J}_{xc}$ on the interacting microstrip resonators can be written in terms of them as follows.

A. Half-Wave Coupled Parallel Rectangular Microstrip Resonators

Let $\tilde{J}_{\substack{z\\x}}^{(1)}(\hat{k}_n,\beta)$ and $\tilde{J}_{\substack{z\\x}}^{(2)}(\hat{k}_n,\beta)$ refer to the Fourier transform of current densities on the resonator to the left of the origin (strip 1) and to the right of the origin (strip 2), respectively. Then, with the shift theorem, we get

$$\tilde{J}_{\substack{z\\x}}^{(1)}(\hat{k}_n,\beta)=e^{-j\hat{k}_n(s+w)}\tilde{J}_{\substack{z\\x}}(\hat{k}_n,\beta) \quad (2)$$

$$\tilde{J}_{\substack{z\\x}}^{(2)}(\hat{k}_n,\beta)=e^{j\hat{k}_n(s+w)}\tilde{J}_{\substack{z\\x}}(\hat{k}_n,\beta). \quad (3)$$

In the even mode, the longitudinal currents on both the strips are equal in magnitude while in the odd mode, they are equal in magnitude but opposite in phase. Taking this into account, we write

$$\tilde{J}_{\substack{zc\\xc}}(\hat{k}_n,\beta)=\left[\pm\delta e^{-j\hat{k}_n(s+w)}+e^{j\hat{k}_n(s+w)}\right]\tilde{J}_{\substack{z\\x}}(\hat{k}_n,\beta) \quad (4)$$

with $\delta=+1$ in even mode, -1 in odd mode, and the positive sign before the quantity δ is taken for the z-directed current and the negative sign is taken for the x-directed current.

B. Quarter-Wave Coupled Parallel Rectangular Microstrip Resonators

The quarter-wave coupled resonator configuration is obtained from the half-wave coupled resonator by introducing a shift of $l/2$ in $+z$ and $-z$ directions in the strip 1 and 2, respectively. Then

$$\tilde{J}_{\substack{z\\x}}^{(1)}(\hat{k}_n,\beta)=e^{j\beta l/2}e^{-j\hat{k}_n(s+w)}\tilde{J}_{\substack{z\\x}}(\hat{k}_n,\beta) \quad (5)$$

$$\tilde{J}_{\substack{z\\x}}^{(2)}(\hat{k}_n,\beta)=e^{-j\beta l/2}e^{j\hat{k}_n(s+w)}\tilde{J}_{\substack{z\\x}}(\hat{k}_n,\beta) \quad (6)$$

and the current densities in the even and odd mode are given by

$$\tilde{J}_{\substack{zc\\xc}}(\hat{k}_n,\beta)=\left[\pm\delta e^{j[\beta l/2-\hat{k}_n(s+w)]}+e^{-j[\beta l/2-\hat{k}_n(s+w)]}\right]\cdot\tilde{J}_{\substack{z\\x}}(\hat{k}_n,\beta) \quad (7)$$

where δ and the sign preceding δ are defined in Section III-A.

In the above expressions, appropriate distributions of the longitudinal and transverse current densities are required. Following the analysis of microstrip resonators presented by Itoh [7], it is prudent to assume basis functions which approximate the actual current distributions. Thus, J_z and J_x for the dominant mode are assumed to have the following form:

$$J_z(x,z)=J_1(x)J_2(z) \quad (8)$$

$$J_x(x,z)=J_3(x)J_4(z) \quad (9)$$

where the functions $J_1(x)$, $J_2(z)$, $J_3(x)$, and $J_4(z)$ appearing above and their transforms are given in [7].

IV. Numerical and Experimental Results

A computer program was developed to evaluate resonant frequencies of interacting rectangular microstrip resonators in the even and odd resonance modes. This program was first verified by comparing the results of a rectangular microstrip resonator with those reported by Itoh [7]. In the computations, the inner products and the roots of the characteristic equation were evaluated with an accuracy up to four significant digits or more. These error criteria are required to exit from integration and root searching routines.

The resonant frequencies of the interacting microstrip resonators were evaluated using the above computer program with appropriate modifications in the current densities corresponding to the physical configurations. In each case, they were evaluated by taking into account both longitudinal and transverse current densities, and compared with those obtained by taking into account longitudinal current density only for various structural parameters. The numerical values were observed to be in good agreement. This means that the effect of the transverse current density for interacting microstrip resonators is negligible as long as microstrip width is much less than the half-wavelength in the dielectric medium. Therefore, the numerical results presented in this paper have been evaluated by taking into account longitudinal current density only.

The experimental resonant frequencies were obtained for microstrip resonators fabricated on an Epsilam-10 substrate ($\epsilon_r=10.2$). They were coupled to a 50-Ω microstrip line at the input and output ports. The capacitive coupling between the microstrip and the resonator was optimized such that the influence of this gap on the resonant frequency was negligible. The even- and odd-mode resonant frequencies were measured in the transmission mode. The numerical and experimental results for each configuration are discussed in the following paragraphs.

A. Half-Wave Coupled Parallel Rectangular Microstrip Resonators

The even- and odd-mode resonant frequencies for the half-wave coupled rectangular resonators have been numerically evaluated with the computer program described above. The results have been obtained for resonators with various normalized widths ($2w/d$), normalized gaps ($2s/d$), and lengths ($2l$). In Fig. 2(a) and (b), the effect of varying $2w/d$ and $2s/d$ is plotted for various resonator lengths while keeping the other parameters fixed. The resonant frequencies in both the modes decrease considerably with $2w/d$ for $2s/d=0.1$. For the fixed value of $2w/d=1.0$, increasing $2s/d$ decreases the even- and odd-mode resonant frequencies only slightly. This behavior may appear somewhat different from what we normally expect, that is, with an increase in $2s/d$, the resonant frequency in the even mode should increase and in the odd mode it should decrease, and finally, it should converge to the resonant frequency of a single microstrip resonator. However, the present behavior is an overall effect since this analysis takes into account the interactions between the resonators as well as the influences of the shielding enclosure. The experimental verification of the resonant frequencies in the even and odd modes is provided in Fig. 2(c) and (d)

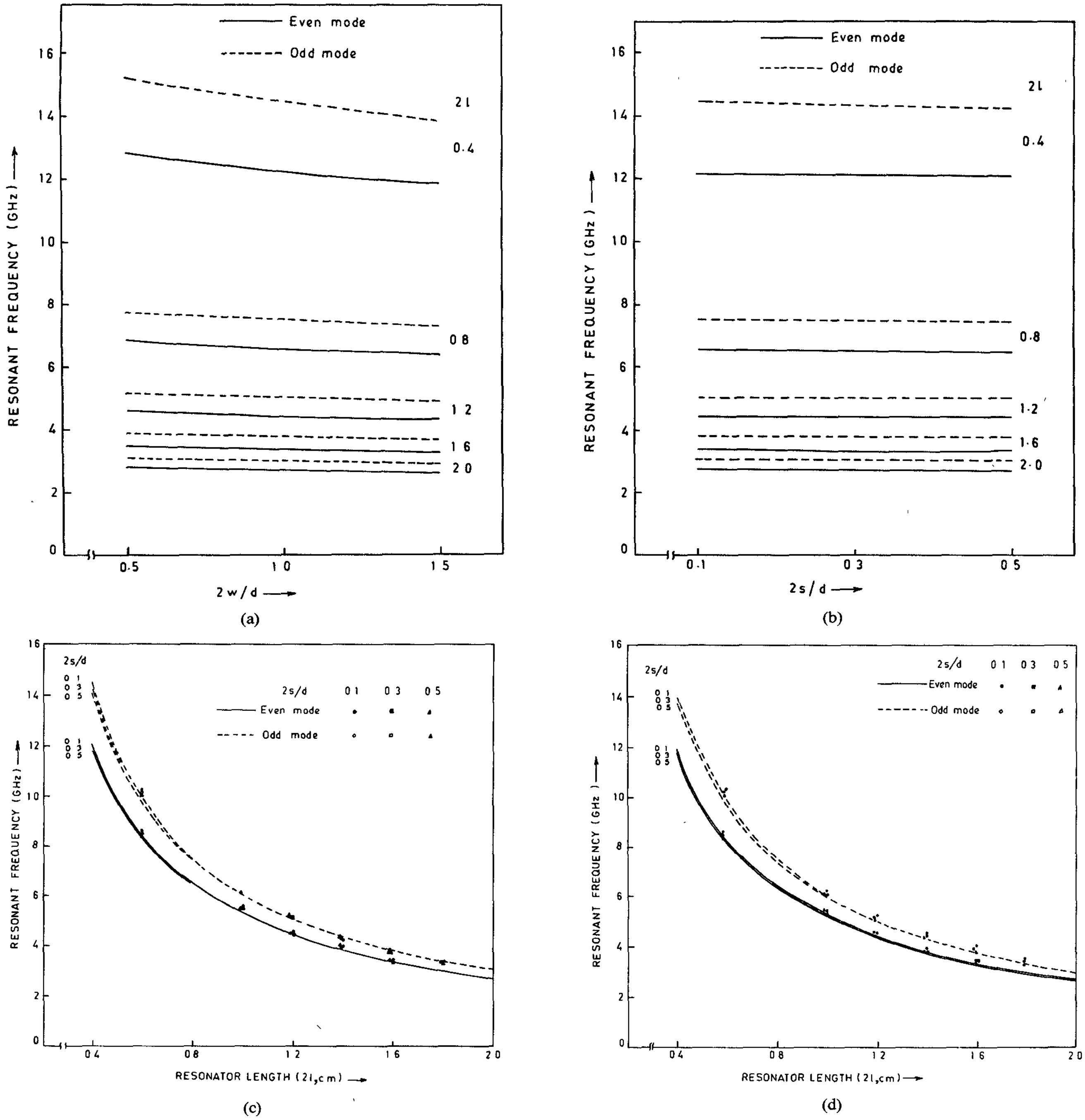

Fig. 2. Resonant frequencies in the even and odd modes for half-wave coupled rectangular microstrip resonators as a function of: (a) $2w/d$ with $2s/d=0.1$; (b) $2s/d$ with $2w/d=1.0$; (c) and (d) resonator length $2l$ with $2w/d=1.0$ and 1.5, respectively. $2a=1.27$ cm; $d+h=1.27$ cm; $d=0.0635$ cm; and $\epsilon_r=10.2$.

for $2w/d=1.0$ and 1.5, respectively, for $2s/d=0.1$, 0.3, and 0.5. These results agree with ± 1.5 percent. The Q-factors of these resonators were observed to be in the range of 100–200.

B. Quarter-Wave Coupled Parallel Rectangular Microstrip Resonators

Similar study on the quarter-wave coupled rectangular resonators show that its resonance behavior is substantially different from that of half-wave coupled rectangular resonators. This is evident from Fig. 3(a) and (b) where the resonant frequencies are plotted against $2w/d$ and $2s/d$, respectively. The difference in the even- and odd-mode resonant frequencies is observed to be more than that of the half-wave coupled resonators. Fig. 3(a) shows that for a fixed value of $2s/d=0.1$, the even-mode resonant frequency increases while the odd-mode resonant frequency decreases with increasing $2w/d$. Similar trend is observed from Fig. 3(b) for resonant frequencies as a function of $2s/d$. These resonant frequencies agree within ± 1.5 percent with the experimental values, as shown in Fig. 3(c).

V. CONCLUSIONS

In this paper, we have presented an analysis of various interacting resonant structures with the full-wave formulation of the spectral domain technique. The resonant frequencies of the half-wave coupled and the quarter-wave coupled rectangular microstrip resonators in the even and odd resonance modes have been

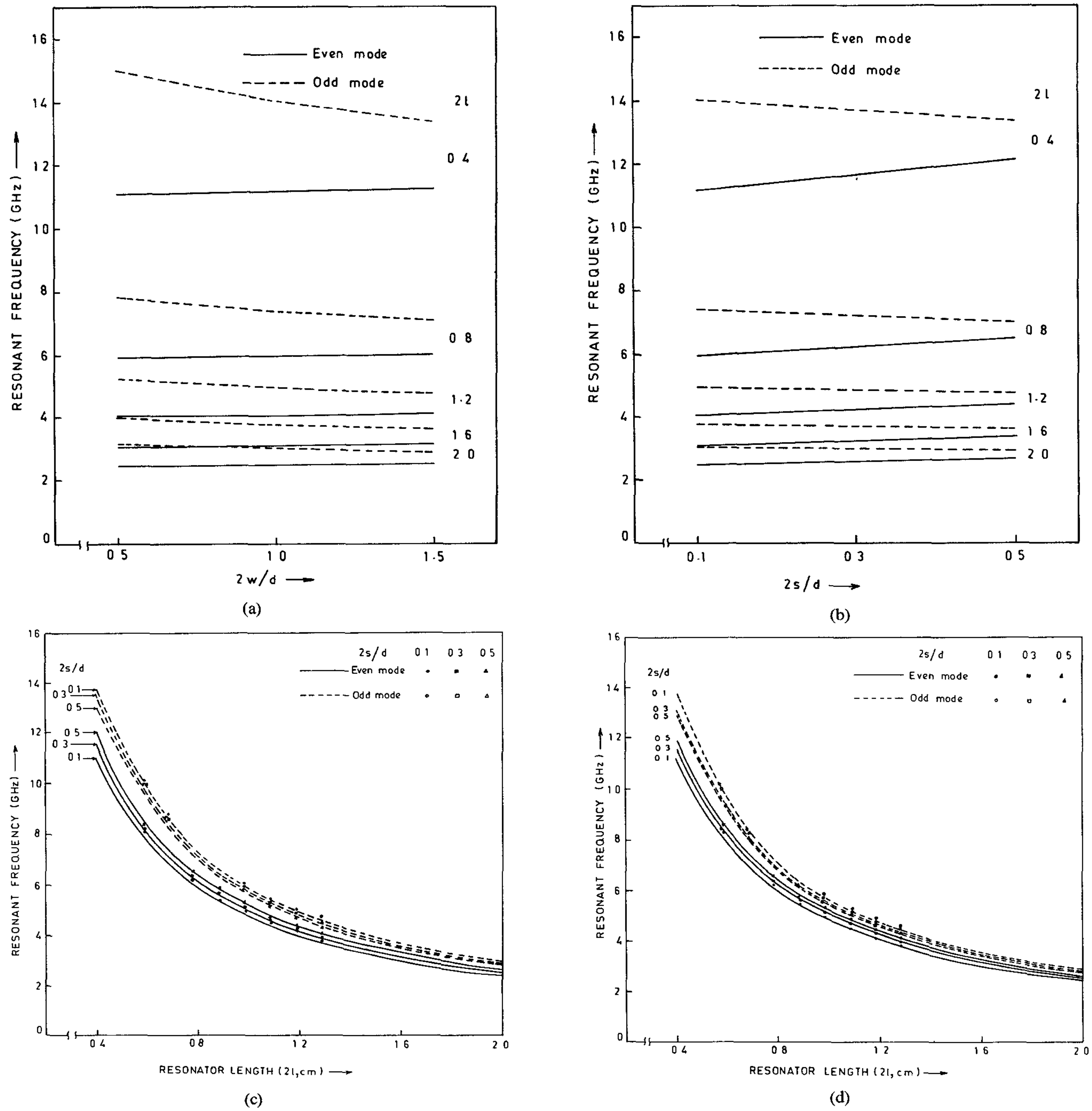

Fig. 3. Resonant frequencies in the even and odd modes for quarter-wave coupled rectangular microstrip resonators as a function of: (a) $2w/d$ with $2s/d = 0.1$; (b) $2s/d$ with $2w/d = 1.0$; (c) and (d) resonator length $2l$ with $2w/d = 1.0$ and 1.5, respectively. $2a = 1.27$ cm; $d + h = 1.27$ cm; $d = 0.0635$ cm; and $\epsilon_r = 10.2$.

evaluated. The numerical results evaluated by taking into account longitudinal current density compare very well with those computed by taking into account both longitudinal and transverse current densities for the structural parameters reported here. The numerical accuracy and efficiency of this technique are achieved by selecting basis functions which approximately represent the actual currents on the resonators. However, the computational accuracy may degrade for small values of $2s$ which corresponds to tightly coupled resonators. This is mainly due to the choice of basis functions used in this investigation. The computer time for a typical computation is about 150 s per structure in each resonance mode on an ICL 1909 (UK) system which is about four to five times slower than an IBM 360 system. The agreement between the theoretical and experimental resonant frequencies has been observed to be within ± 1.5 percent. This shows that the divergence in the even- and odd-mode phase velocities can be accurately predicted with the present analysis which takes into account the interactions between the resonant structures as well as the influences of the shielding waveguide.

References

[1] G. I. Zysman and A. K. Johnson, "Coupled transmission line networks in an inhomogeneous medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 753–759, Oct. 1969.

[2] F. C. de Ronde, "Recent development in broadband directional couplers on microstrip," in *1972 IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 215–217.

[3] B. Shelag and B. E. Spielman, "Broadband directional couplers using microstrip and dielectric overlays," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 1216–1220, Dec. 1974.

[4] A. E. Ross and D. Pompei, "Improvement of performances of microstrip structures by equalization of phase velocities," in *1978 IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 41–43.
[5] B. Easter and J. G. Richings, "Effects associated with radiation in coupled half-wave open circuit microstrip resonators," *Electron. Lett.*, vol. 8, pp. 198–199, June 1972.
[6] R. H. Jansen, "Computer analysis of edge coupled planar structures," *Electron. Lett.*, vol. 10, pp. 520–522, Nov. 1974.
[7] T. Itoh, "Analysis of microstrip resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 946–952, Nov. 1974.
[8] A. K. Sharma and B. Bhat, "Spectral domain analysis of discontinuity microstrip structures," presented at 1981 National Radio Science Meeting (Los Angeles, CA), June 15–19, 1981.
[9] A. K. Sharma and B. Bhat, "Analysis of interacting rectangular microstrip resonators," presented at Int. Electrical, Electronics Conf. and Expos. (Toronto, Canada), Oct. 5–7, 1981.
[10] A. K. Sharma and B. Bhat, "Analysis of microstrip resonant structures," presented at Int. Symp. Microwaves and Communication (Kharagpur, India), Dec. 29–30, 1981, paper MN 3.7.
[11] A. K. Sharma, "Spectral domain analysis of microstrip resonant structures," Ph.D. Thesis, Indian Institute of Technology, Delhi, December 1979.

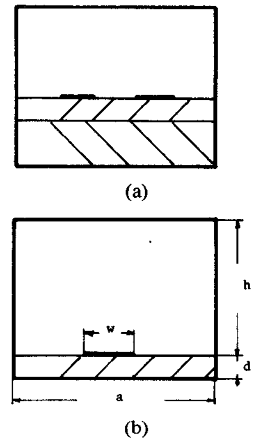

Fig. 1. (a) Cross-section view of planar MIC structure analyzable by the computer program of [4]. (b) Shielded microstrip line.

An Accurate Bivariate Formulation for Computer-Aided Design of Circuits Including Microstrip

Y. L. CHUA, J. B. DAVIES, AND D. MIRSHEKAR-SYAHKAL

Abstract —**An accurate and fast bivariate interpolation technique is used to compute the microstrip parameters at an arbitrary frequency and of any strip width. This technique allows computation of the effective dielectric constant, characteristic impedance, dielectric loss, and the conductor loss of microstrip in a time appropriate for computer-aided design application. By combining interpolation techniques with a highly accurate theory, computing is more accurate or faster than earlier theories, which achieve speed of computation by *a priori* approximations.**

I. Introduction

Various theories exist for microstrip and related planar lines which are 'exact-in-the-limit.' Properly implemented, these result in a computer program where for any analysis the designer can choose between an approximate, cheap result and an accurate, expensive result, the cost being measured in computer time and possibly computer storage. Similar to the approximate, cheap result, is the use of an *a priori* approximate theory, such as the many quasi-TEM theories, and approximate frequency-dependent theories [1], [2]. However, for interactive computer-aided design (CAD), and on other occasions, the time or cost of the accurate results may not be acceptable, and the designer has to make an awkward compromise.

The purpose of this paper is to show how the accurate results of a microstrip analysis program [3], [4] can be used to provide the data base for a subsequent program which in turn can give accurate and fast results over some specified range of parameters, such as frequency and strip width. It involves essentially bivariate interpolation over specified ranges, and as such, the method can be applied to many two-parameter problems. In this paper, the technique is illustrated with the accurate evaluation of four microstrip characteristics (phase velocity, characteristic impedance, attenuation due to conductor losses, and dielectric losses). Based on a published computer program [4], the computing times are reduced by a factor of a hundred or more in calculating these four characteristics over a continuous range of frequencies and of strip width.

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II. Theory

The theory of this paper is applied and illustrated with just the one basic computer program, one for the accurate analysis of microstrip, but its application to similar programs is implicit.

Reference [4] describes a program that considers the cross sections of Fig. 1(a), and for either single or coupled microstrip calculates the effective dielectric constant ϵ_{eff} (or phase velocity). Then, if required, it calculates the characteristic impedance Z_c, attenuation due to imperfect conductor α_c, and attenuation due to imperfect dielectric α_d.

Though the program is efficient in its class, it is still slow for CAD purposes. If results of the program ϵ_{eff}, Z_c, α_c, and α_d are obtained over the range of specified strip widths w and frequencies f, sets of this data can be considered as bivariate functions of w and f. We assume that the designer has control of, or needs results of, continuous parameters w and f, whereas the dielectric thickness and permittivity are of discrete values dictated by the substrate manufacturer. Other parameters affecting the results are the height and width of the conducting enclosure. These can be fixed at certain acceptable dimensions, possibly large enough to have negligible influence on ϵ_{eff}, etc.

Having generated the data sets, the objective is to find a suitable bivariate interpolation scheme which can accurately and efficiently give the values of ϵ_{eff}, Z_c, α_c, and α_d at any (w, f) values—not just at the data set points. Since accuracy and high efficiency are prerequisites for the interpolating method, the 'spline' technique using a 'tensor product' algorithm has been found to fulfill the requirements [5]. There are other methods for interpolation in one dimension, but their effectiveness for two-dimensional problems is subject to dispute [6].

Spline interpolation by means of the basis-spline function is a relatively new technique. Evolved through research on piecewise polynomial interpolation, it has gained importance in numerical analysis. It is widely used in computer graphic software, where extra smoothness, fast system response, and good interpolating accuracy are needed [7]. To interpolate with spline functions, details can be found in [5], [7]. However, a brief account seems appropriate for its use for the microstrip line.

We consider the one-dimensional spline technique first, the

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